Theoretical status of $|V_{xb}|$

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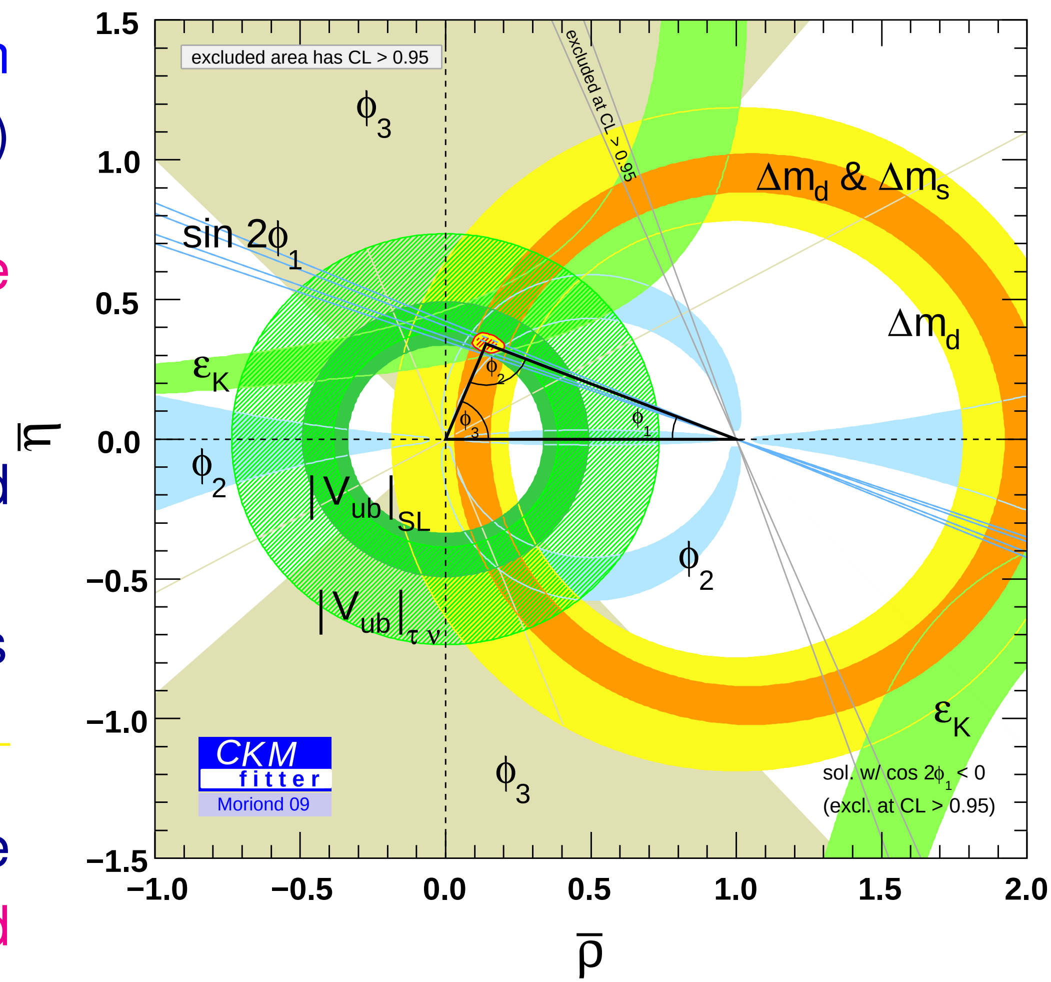
Beauty 2009, Heidelberg, Sept 7–11

The importance of $|V_{cb}|$ and $|V_{ub}|$

- $|V_{cb}|$: large part of the uncertainty in ϵ_K constraint and $K \to \pi\nu\bar{\nu}$ ($\propto |V_{cb}|^4$)

- $|V_{ub}|$: dominant uncertainty of the side opposite to $\beta \equiv \phi_1$

  Look for NP: compare (i) angles and sides; (ii) tree and loop processes ... semileptonic decays crucial for this



- The level of agreement between the measurements is often misinterpreted

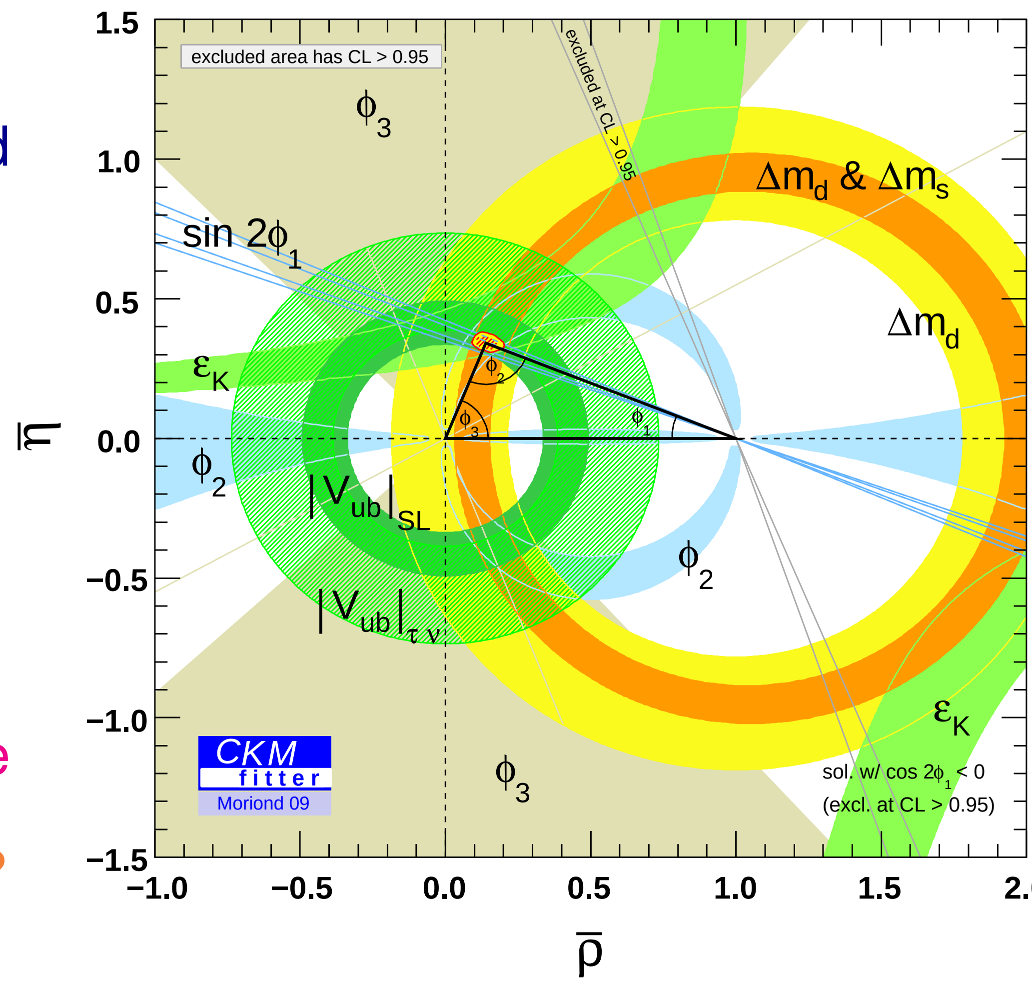
- 10–20% non-SM contributions to most loop-mediated transitions are still possible

Determination of $|V_{ub}|$ is far from settled

- Determined by tree-level decays
  Crucial for comparing tree-dominated and loop-mediated processes

- $|V_{ub}|_{\pi\ell\bar{\nu}-\mathrm{LQCD}} = (3.5 \pm 0.5) \times 10^{-3}$
  $|V_{ub}|_{\mathrm{incl-BLNP}} = (4.06 \pm 0.40) \times 10^{-3}$
  $|V_{ub}|_{\mathrm{incl-BLL}} = (4.87 \pm 0.45) \times 10^{-3}$
  $|V_{ub}|_{\tau\nu} = (5.2 \pm 0.5 \pm 0.4_{f_B}) \times 10^{-3}$
  SM CKM fit $(\sin 2\beta)$ favors small value

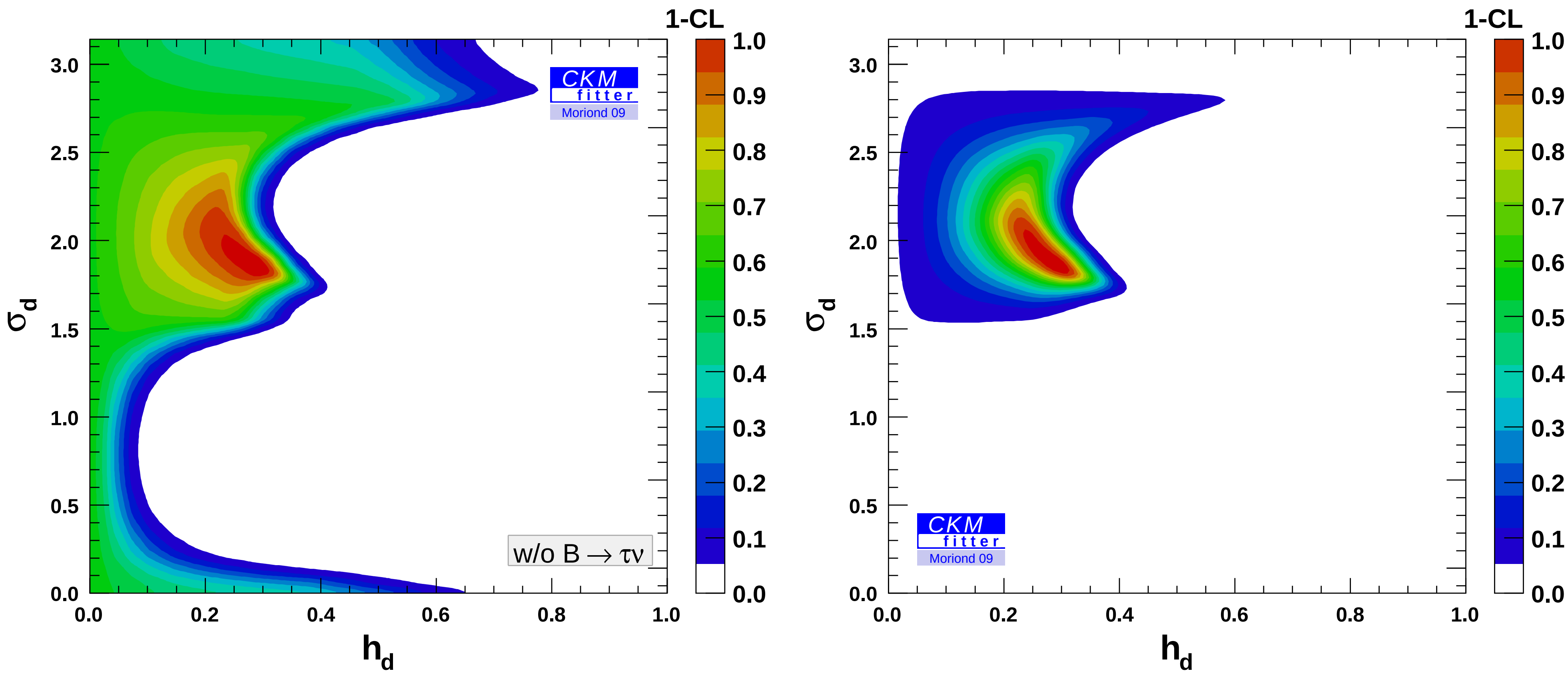
- Fluctuation, bad theory, new physics?



- $b \to q\gamma,\ q\,\ell^+\ell^-,\ q\,\nu\bar{\nu}\ \ (q = s, d)$ are sensitive probes of the SM; theoretical tools same as for $|V_{ub}|$ — accuracy of theory limits sensitivity to NP

$|V_{ub}|$ and new physics ...

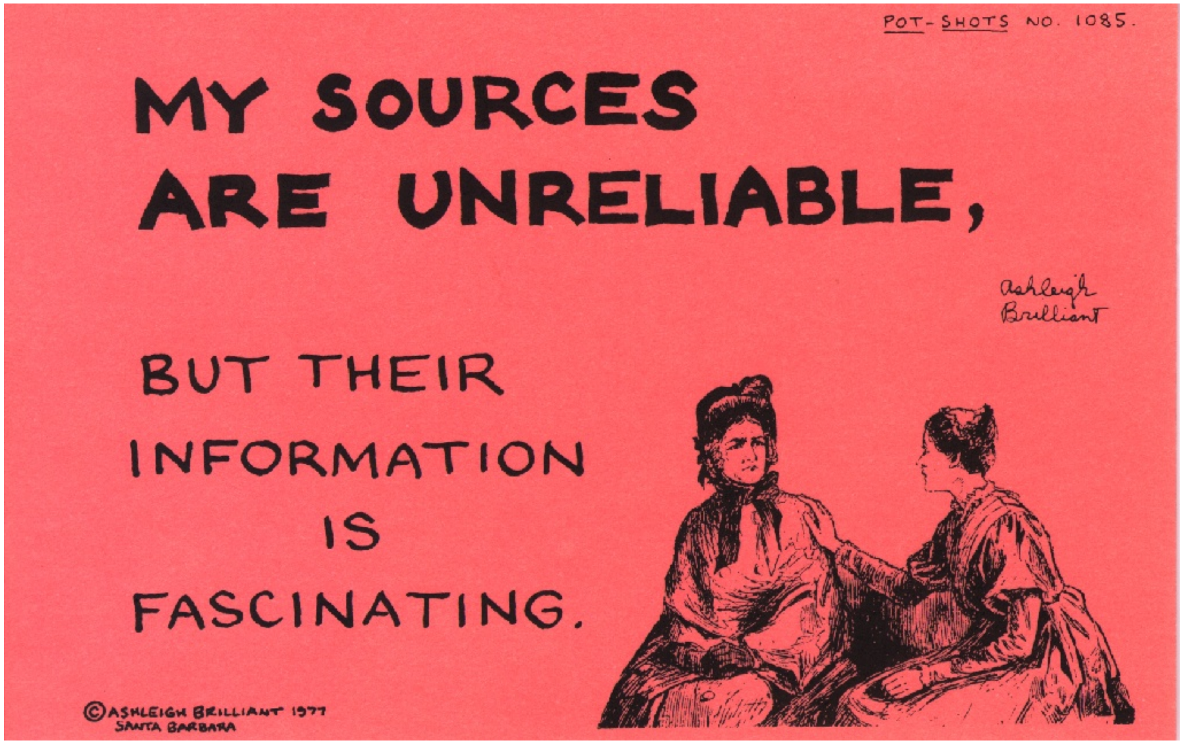
- At the present time, including $B \to \tau\bar{\nu}$, the SM is "disfavored" at $> 2\sigma$



Parameterize NP in B^0–$\overline{B}^0$ mixing: $M_{12} = M_{12}^{\rm SM}\,(1 + h_d\, e^{2i\sigma_d})$

- There are NP models that would ease this "tension" (that's not the real question)

The name of the game



The SM shows impressive consistency — even by Stockholm standards

Only robust deviations from model independent theory are likely to be interesting

Outline

- $|V_{cb}|$ from exclusive and inclusive decays [not much new conceptually on the theory side]
- Exclusive $b \to u\ell\bar{\nu}$ very briefly
- Towards a complete description of $B \to X_u\ell\bar{\nu}$
  - Brief discussion of $B \to X_s\gamma$ [ZL, Stewart, Tackmann, arXiv:0807.1926]
  - A glimpse at SIMBA [Bernlochner, Lacker, ZL, Stewart, Tackmann, Tackmann, to appear]
- Conclusions

Disclaimers: have to skip many interesting topics in 25 min [my apologies for missing refs]
e.g.: Lattice QCD, form factor bounds,
concentrate on some recent developments, mostly related to $b \to u$

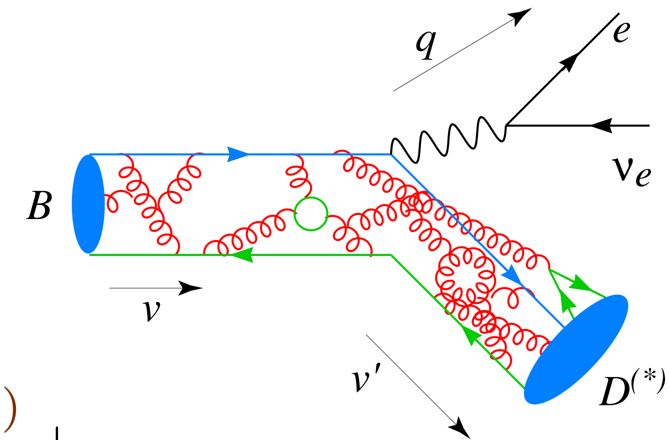
Determination of $|V_{cb}|$

$|V_{cb}|$ from $B \to D^{(*)}\ell\bar\nu$

- Heavy Quark Symmetry: brown muck only feels $v \to v'$ (not $m_b \to m_c$ or $\vec{s}_b \to \vec{s}_c$)

$$\frac{\mathrm{d}\Gamma(B \to D^{(*)}\ell\bar\nu)}{\mathrm{d}w} = (\dots)\,(w^2-1)^{3(1)/2}\,|V_{cb}|^2\,\mathcal{F}^2_{(*)}(w)$$

$w \equiv v \cdot v'$ Isgur-Wise function $+ \dots$



$$\mathcal{F}(1) = 1_{\text{Isgur-Wise}} + 0.02_{\alpha_s,\alpha_s^2} + \frac{\text{(lattice or models)}}{m_{c,b}} + \dots$$

$$\mathcal{F}_*(1) = 1_{\text{Isgur-Wise}} - 0.04_{\alpha_s,\alpha_s^2} + \frac{0_{\text{Luke}}}{m_{c,b}} + \frac{\text{(lattice or models)}}{m_{c,b}^2} + \dots$$

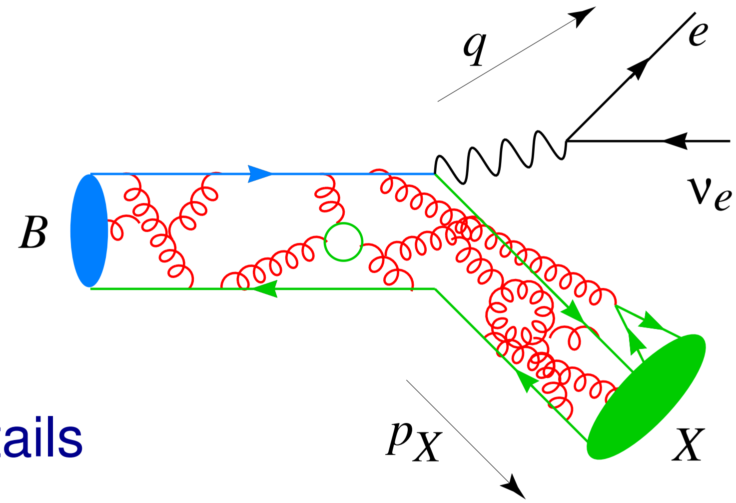
- Lattice QCD: $\mathcal{F}_*(1) = 0.921 \pm 0.024$, $\mathcal{F}(1) = 1.074 \pm 0.024$ [arXiv:0808.2519, hep-lat/0409116]
- Need constraints on shape to fit [Boyd, Grinstein, Lebed; Caprini, Lellouch, Neubert]
- Need some understanding of decays to higher mass X_c states (backgrounds)
- Data: $|V_{cb}\,\mathcal{F}_*(1)| = (35.75 \pm 0.42) \times 10^{-3}$, $|V_{cb}\,\mathcal{F}(1)| = (42.3 \pm 1.5) \times 10^{-3}$ [HFAG]
  [note: χ^2/dof $= 39.6/21\ (56.9/21)$, CL $= 0.8\%\ (4E{-}5)$]

Why inclusive decays?

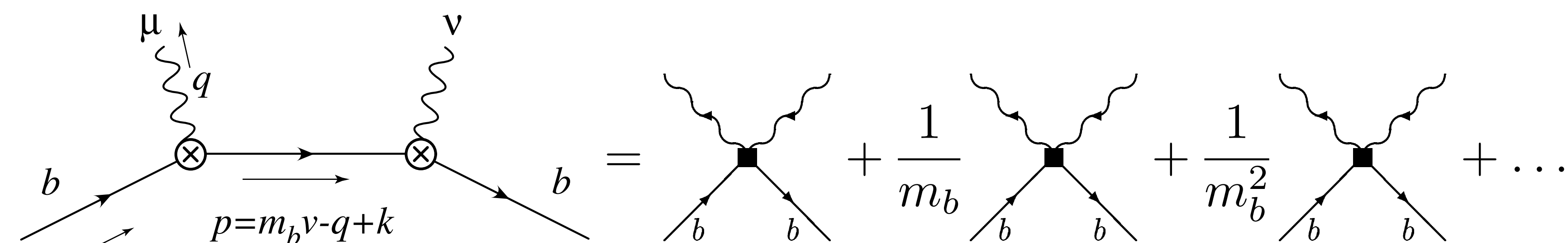
- Sum over hadronic final states, subject to constraints determined by short distance physics

  Decay: short distance (calculable)

  Hadronization: long distance (nonperturbative), but probability to hadronize is unity; sum over details



- Combine operator product expansion (OPE) and heavy quark symmetry



$\sim$ field theoretic version of multipole expansion

Can think of the OPE as expansion of forward scattering amplitude in $k \sim \Lambda_{\rm QCD}$

$|V_{cb}|$ from inclusive decays

- Rates calculable in an OPE, expansion in $\Lambda_{\rm QCD}/m_b$ and $\alpha_s(m_b)$:

$$d\Gamma = \begin{pmatrix} b\text{ quark} \\ \text{decay} \end{pmatrix} \times \left\{ 1 + \frac{0}{m_b} + \frac{f(\lambda_1, \lambda_2)}{m_b^2} + \ldots + \alpha_s(\ldots) + \alpha_s^2(\ldots) + \ldots \right\}$$

In "most" of the phase space, details of b quark wavefunction unimportant, only averages matter: $\lambda_1 \sim \langle k^2 \rangle$, $\lambda_2 \sim \langle \sigma_{\mu\nu} G^{\mu\nu} \rangle = (m_{B^*}^2 - m_B^2)/4$, ...

Must use a short distance scheme to get well behaved expansions in α_s

- $|V_{cb}|$ & hadronic param's (m_b, $\lambda_{1,2}$, ...) fitted to ~ 100 observables; also tests theory

- $1S$ scheme: $|V_{cb}| = (41.56 \pm 0.39 \pm 0.08) \times 10^{-3}$, $m_b^{1S} = 4.75 \pm 0.06\,\text{GeV}$ [HFAG 2007]

  waiting for updated HFAG fit

- kinetic scheme: $|V_{cb}| = (41.31 \pm 0.49 \pm 0.08 \pm 0.58) \times 10^{-3}$ [HFAG 2009]

  n.b.: not all expressions public, we couldn't reproduce some

Some comments on $|V_{cb}|$

- Role of $B \to X_s\gamma$ spectrum: including moments lowers m_b by $\sim 50\,\mathrm{MeV}$
  - No significant improvement for $|V_{cb}|$
  - To pin down m_b to use for $|V_{ub}|$, there is a better way to include $B \to X_s\gamma$ data

  Values on previous page obtained from $B \to X_c\ell\bar\nu$ only

- Theory improvements
  - Extending $\alpha_s^2\beta_0$ to α_s^2: shifts $|V_{cb}|$ by -0.3% in $1S$ scheme [Melnikov; Czarnecki, Pak]
  - $\Lambda_{\mathrm{QCD}}^4/m_b^4$: small effect ($\sim 0.25\%$) on total rate [Dassinger, Turczyk, Mannel]
  - $\alpha_s\,\lambda_1/m_b^2$: expect $\sim 20\%$ in fitted λ_1 value, small effect on $|V_{cb}|$ [Becher, Boos, Lunghi]
  - $\alpha_s\,\lambda_2/m_b^2$: being calculated... possibly most important beyond what's included

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- $|V_{cb}|_{\mathrm{incl.}}$ is about 8% (2σ) higher than $|V_{cb}|_{\mathrm{excl.}}$

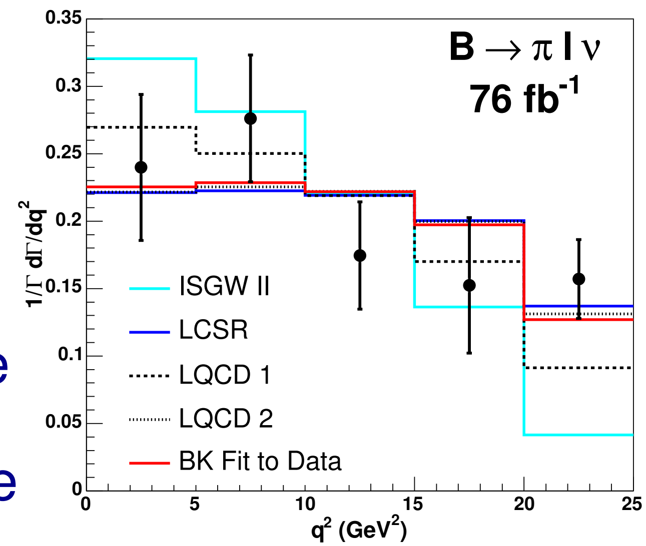
- Hard to imagine how theory input for $|V_{cb}|_{\mathrm{incl.}}$ could change substantially

Determination of $|V_{ub}|$

$|V_{ub}|$ from exclusive decays

- Less constraints from heavy quark symmetry than in $B \to D^{(*)}\ell\bar{\nu}$
- $B \to \ell\bar{\nu}$: measures $f_B \times |V_{ub}|$ — need f_B (lattice QCD)
- $B \to \pi\ell\bar{\nu}$:

$$\frac{\mathrm{d}\Gamma(\overline{B}^0 \to \pi^+\ell\bar{\nu})}{\mathrm{d}q^2} = \frac{G_F^2 |\vec{p}_\pi|^3}{24\pi^3} \, |V_{ub}|^2 \, |f_+(q^2)|^2$$



Determination of $f_+(q^2)$ in the hands of lattice QCD

... under better control at large q^2 (small $|\vec{p}_\pi|$);

... calculations in larger q^2 range may become possible

Continuum theory input: analyticity constraint on shape (needs a few $f_+(q^2)$ values)

- $B \to \rho\ell\bar{\nu}$ is harder (sizable Γ_ρ/m_ρ), $B \to \eta^{(\prime)}\ell\bar{\nu}$ is even harder
- Expect $B \to \pi\ell\bar{\nu}$ to dominate $|V_{ub}|_{\mathrm{excl}}$ for the foreseeable future

“Grinstein-type double ratios”

- Continuum theory may be competitive using HQS + chiral symmetry suppression
- $\dfrac{f_B}{f_{B_s}} \times \dfrac{f_{D_s}}{f_D}$ — lattice: double ratio $= 1$ within few $\%$ [Grinstein ’93]
- $\dfrac{f^{(B\to\rho\ell\bar\nu)}}{f^{(B\to K^*\ell^+\ell^-)}} \times \dfrac{f^{(D\to K^*\ell\bar\nu)}}{f^{(D\to\rho\ell\bar\nu)}}$ or q^2 spectra — accessible soon? [ZL, Wise; Grinstein, Pirjol]

  CLEO-c $D \to \rho\ell\bar\nu$ data still consistent with no $SU(3)$ breaking in form factors [ZL, Stewart, Wise]

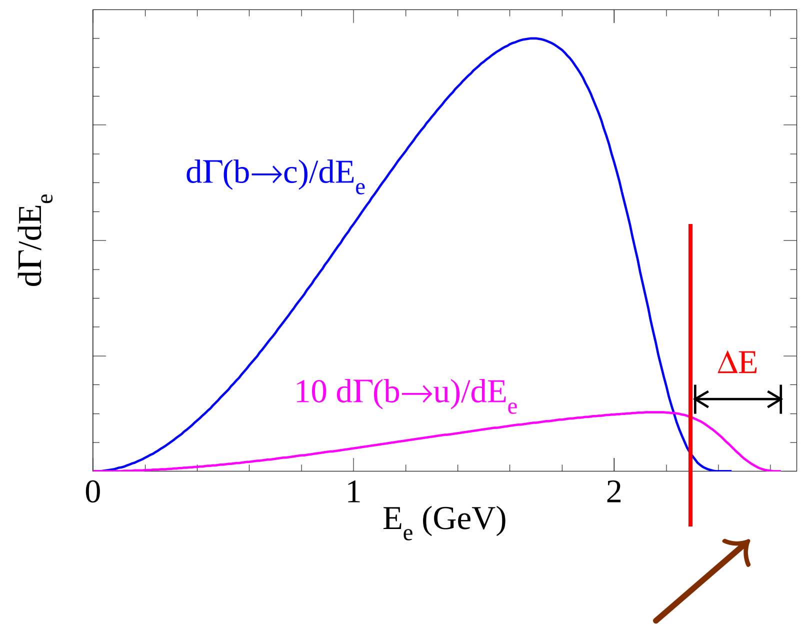
  Could lattice QCD do more to pin down the corrections?

  Worth looking at similar ratio with K, π — role of B^* pole...?
- $\dfrac{\mathcal{B}(B \to \ell\bar\nu)}{\mathcal{B}(B_s \to \ell^+\ell^-)} \times \dfrac{\mathcal{B}(D_s \to \ell\bar\nu)}{\mathcal{B}(D \to \ell\bar\nu)}$ — very clean... after 2015? [Ringberg workshop, ’03]
- $\dfrac{\mathcal{B}(B_u \to \ell\bar\nu)}{\mathcal{B}(B_d \to \mu^+\mu^-)}$ — even cleaner... ever possible? [Grinstein, CKM’06]

The challenge of inclusive $|V_{ub}|$ measurements

- Total rate predicted with $\sim 4\%$ accuracy, similar to $\mathcal{B}(B \to X_c \ell \bar{\nu})$ [Hoang, ZL, Manohar]
- To remove the huge charm background $(|V_{cb}/V_{ub}|^2 \sim 100)$, need phase space cuts

  Can enhance pert. and nonpert. corrections



- Instead of being constants, the hadronic parameters become functions (like PDFs)

  Leading order: universal & related to $B \to X_s\gamma$;
  $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$: several new unknown functions

  Nonperturbative effects shift endpoint $\frac{1}{2}\,m_b \to \frac{1}{2}\,m_B$ & determine its shape

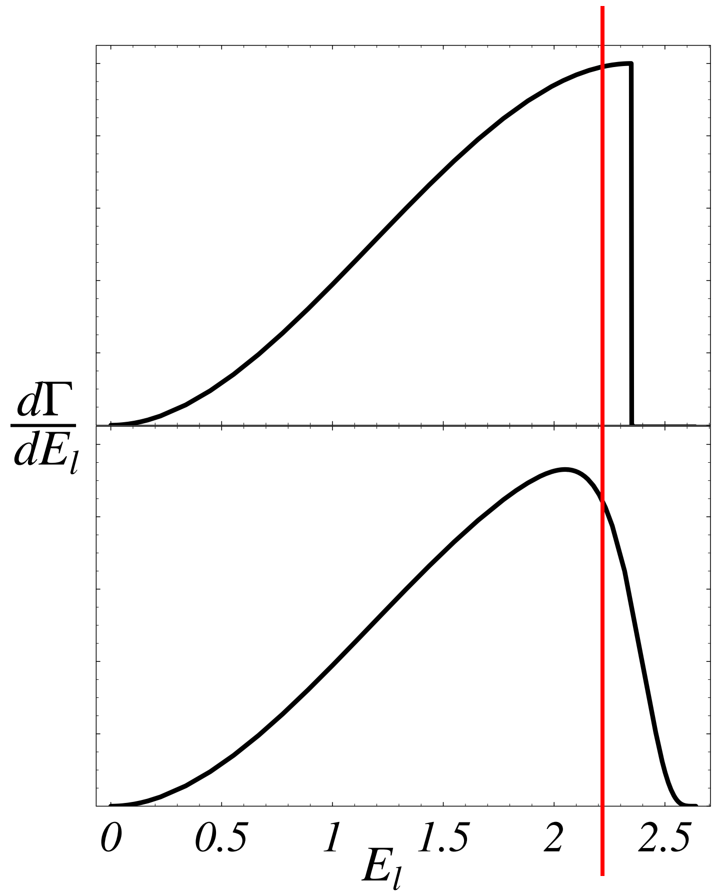
- Endpoint region determined by b quark PDF in B; want to extract it from data to make predictions — at lowest order $\propto B \to X_s\gamma$ photon spectrum

  [Bigi, Shifman, Uraltsev, Vainshtein; Neubert]

Shape function: lepton endpoint vs. $B \to X_s\gamma$

b quark decay spectrum

with a model for b quark PDF



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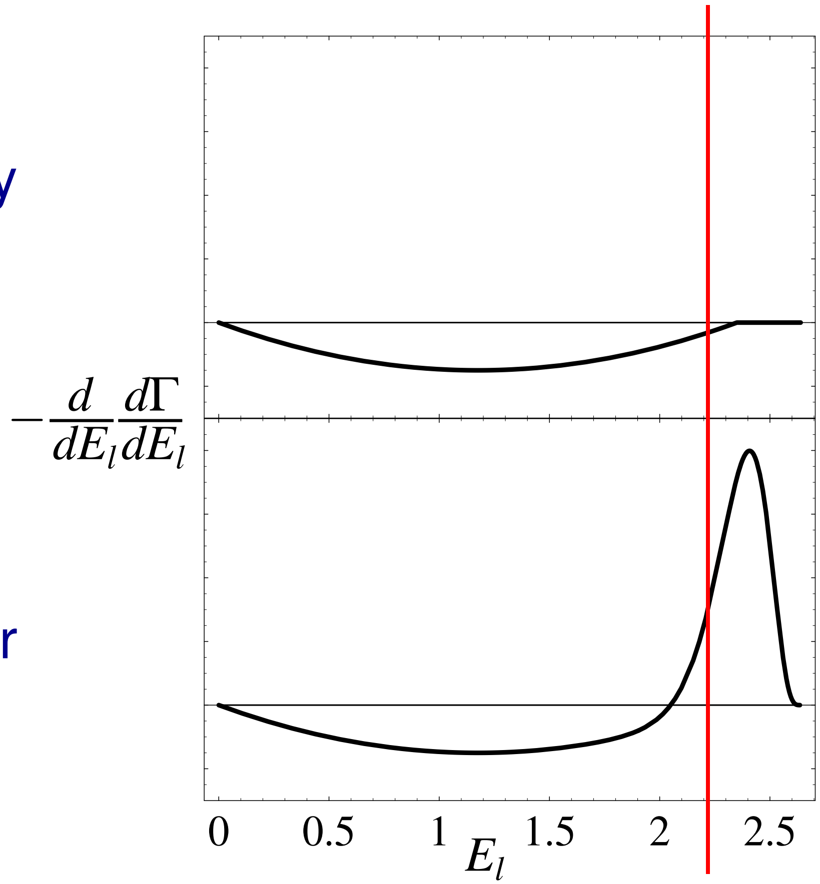

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Shape function: lepton endpoint vs. $B \to X_s\gamma$

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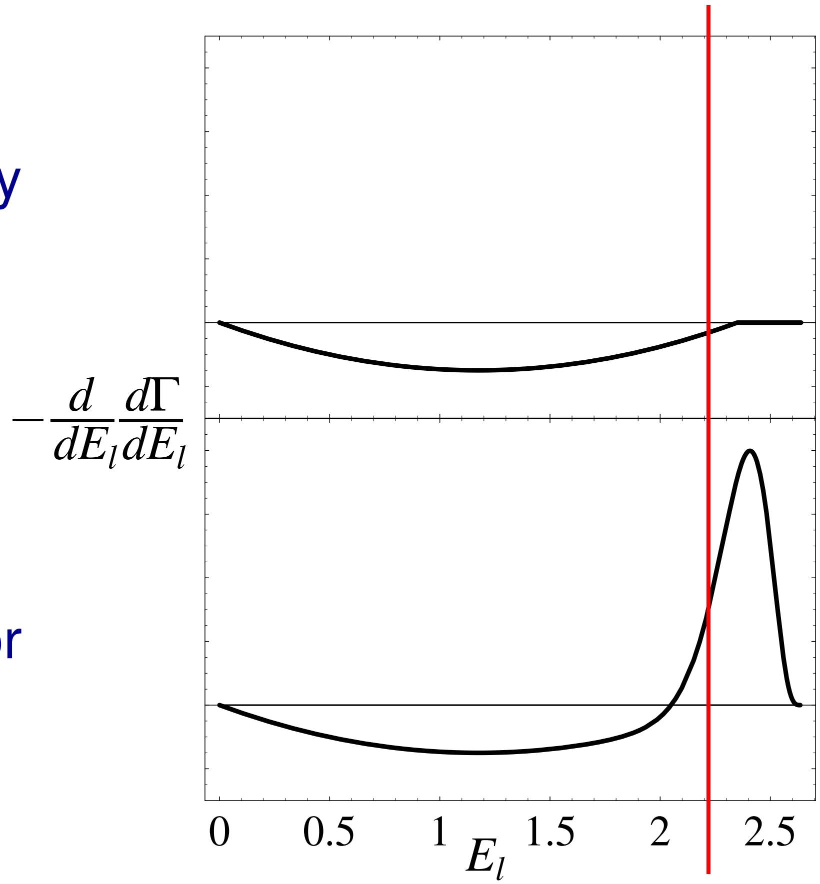
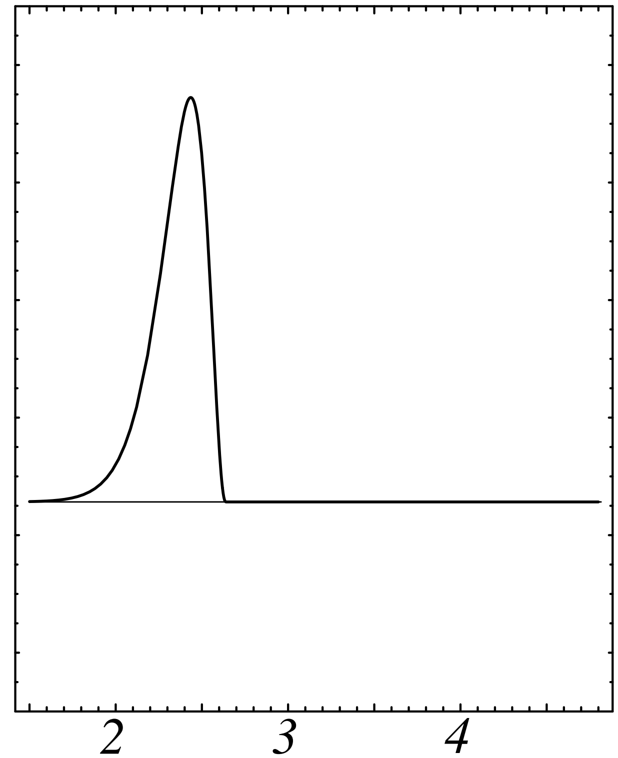


difference:



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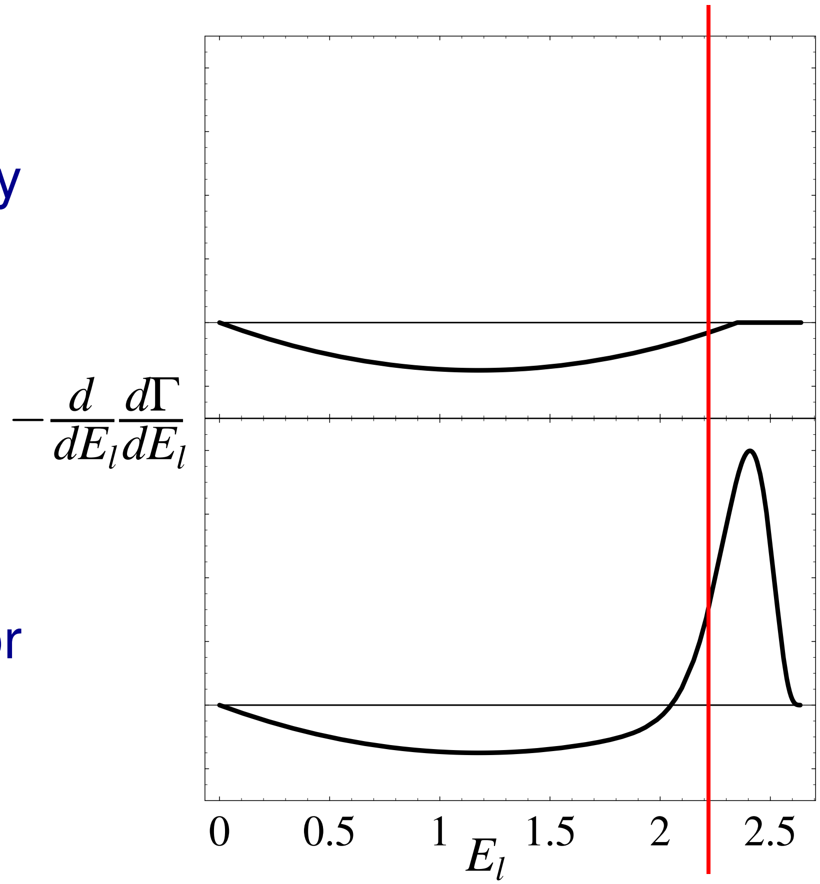

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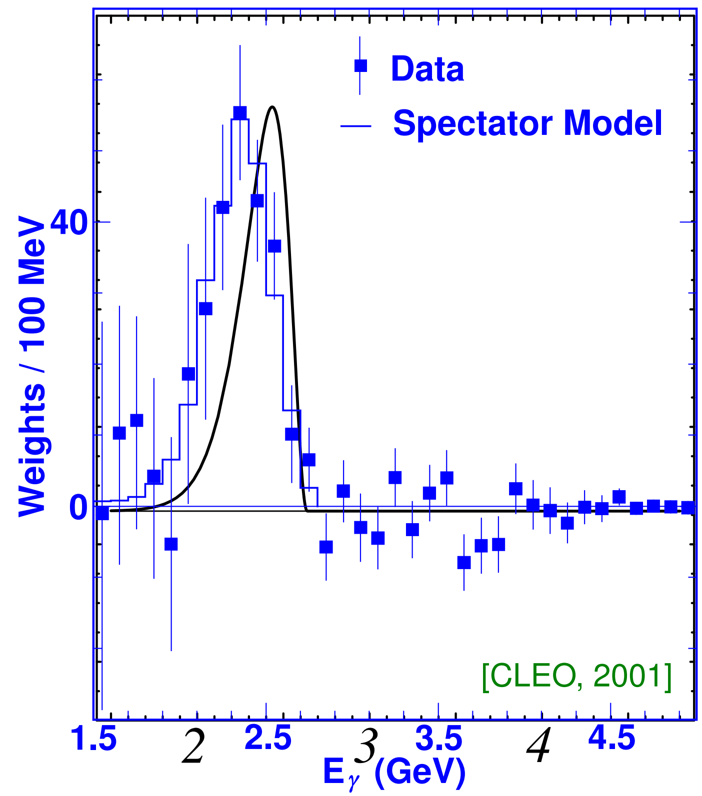

Shape function: lepton endpoint vs. $B \to X_s \gamma$

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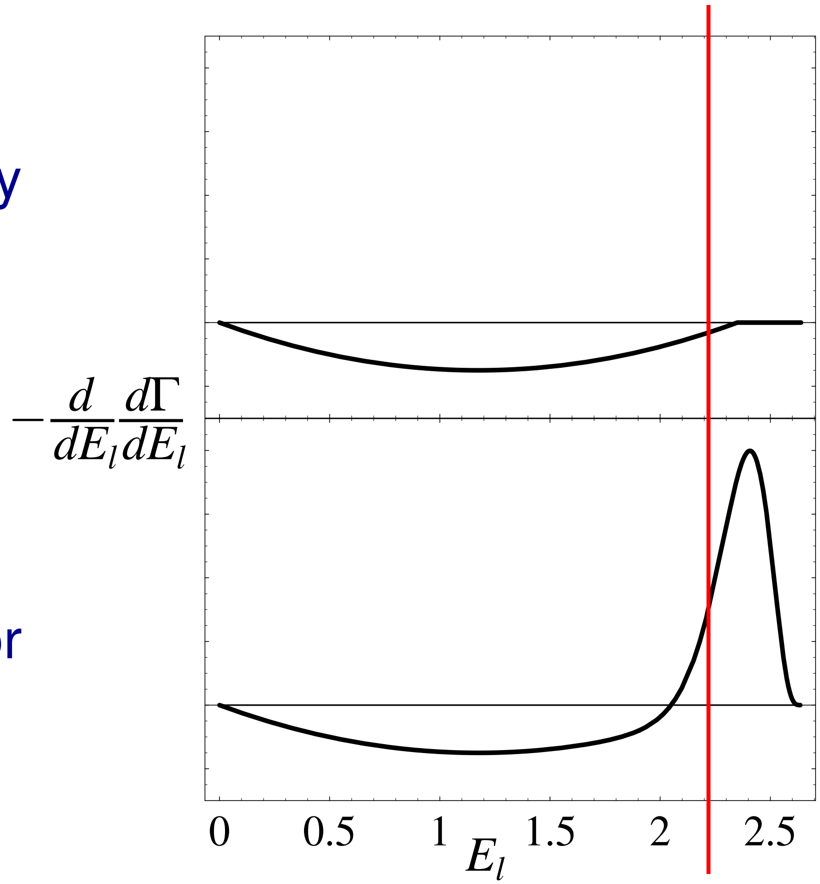

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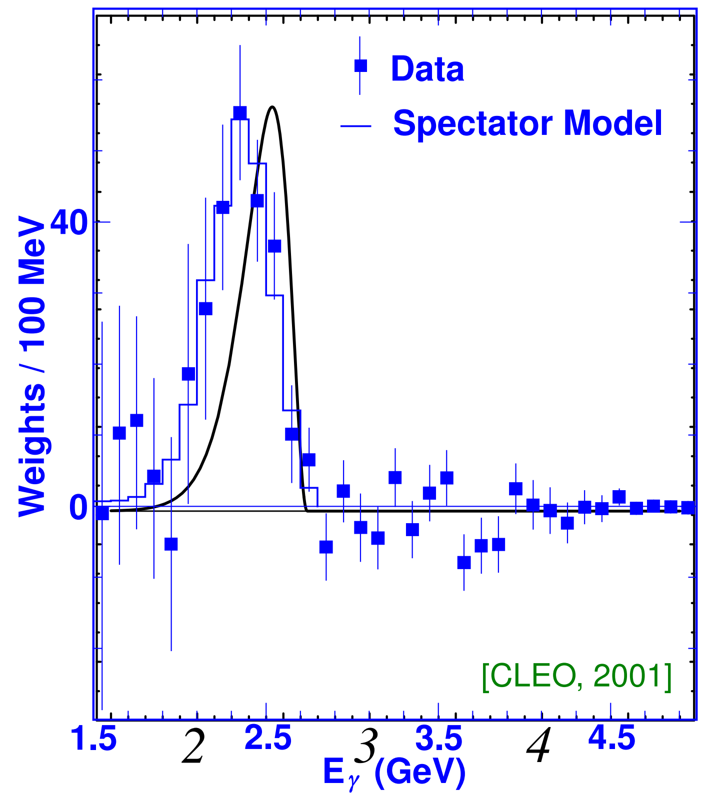


- Both of these spectra determined at lowest order by the b quark PDF in B meson
- Lots of work toward extending beyond leading order; some open issues remain...

Regions of $B \to X_s\gamma$ phase space

- Important both for $|V_{ub}|$ and constraining NP
- $p_X^+ \equiv m_B - 2E_\gamma \lesssim 2\,\mathrm{GeV}$, and $< 1\,\mathrm{GeV}$ at the peak

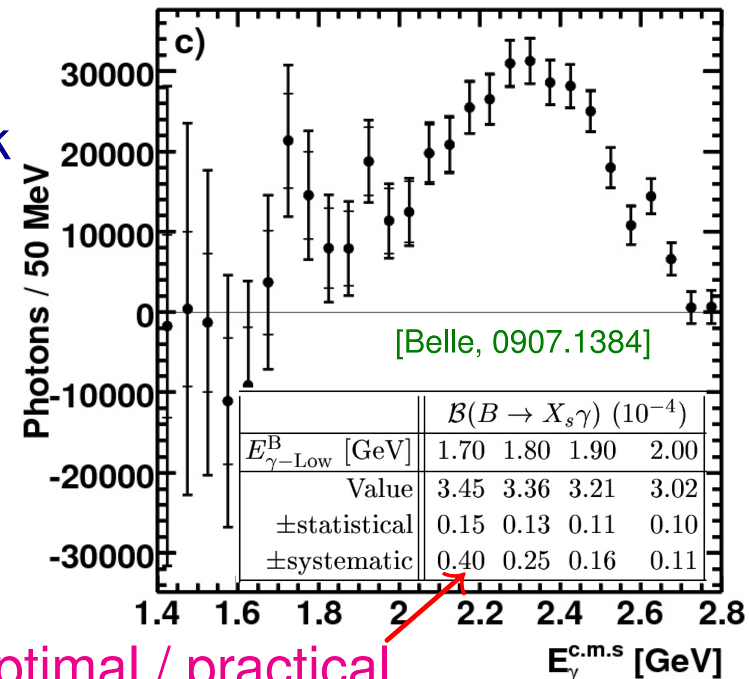
  Three cases: 1) $\Lambda_{\rm QCD} \sim m_B - 2E_\gamma \ll m_B$

  2) $\Lambda_{\rm QCD} \ll m_B - 2E_\gamma \ll m_B$

  3) $\Lambda_{\rm QCD} \ll m_B - 2E_\gamma \sim m_B$

  Neither 1) nor 2) is fully appropriate

  [Sometimes called: 1) SCET and 2) MSOPE regions]



	$\mathcal{B}(B \to X_s\gamma)$ (10^{-4})			
$E^{\rm B}_{\gamma-{\rm Low}}$ [GeV]	1.70	1.80	1.90	2.00
Value	3.45	3.36	3.21	3.02
±statistical	0.15	0.13	0.11	0.10
±systematic	0.40	0.25	0.16	0.11

- Not clear if reducing $E_\gamma^{\rm cut}$ to $\sim 1.7\,\mathrm{GeV}$ is indeed optimal / practical

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- Included all known results in regions 1) – 2) [ZL, Stewart, Tackmann, 0807.1926]

LL:	1–loop $\Gamma_{\rm cusp}$,	tree-level matching	
NLL:	2–loop $\Gamma_{\rm cusp}$,	1–loop matching,	1–loop γ_x
NNLL:	3–loop $\Gamma_{\rm cusp}$,	2–loop matching,	2–loop γ_x

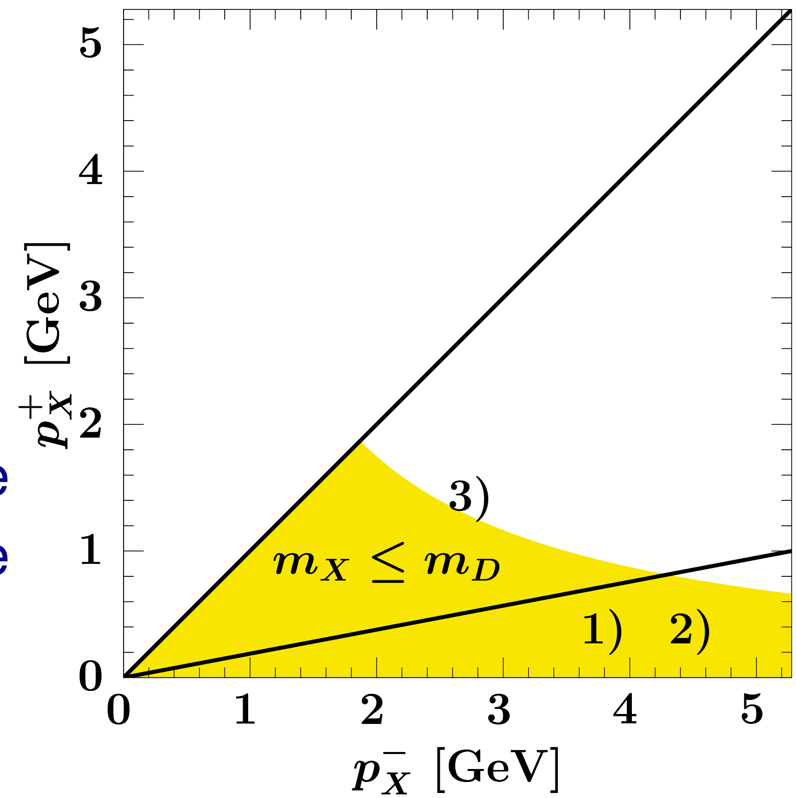
Showed how to combine 1)-2) w/o expanding shape function in Λ/p_X^+

Regions of $B \to X_u \ell \bar{\nu}$ phase space

- "Natural" kinematic variables: $p_X^\pm = E_X \mp |\vec{p}_X|$ — "jettyness" of hadronic final state
  $B \to X_s \gamma$: $p_X^+ = m_B - 2E_\gamma$ & $p_X^- \equiv m_B$, but independent variables in $B \to X_u \ell \bar{\nu}$

- Three cases: 1) $\Lambda \sim p_X^+ \ll p_X^-$ } SF region
  2) $\Lambda \ll p_X^+ \ll p_X^-$ } SF region
  3) $\Lambda \ll p_X^+ \sim p_X^-$ local OPE region

  Want to make no assumptions how p_X^- compares to m_B



- $B \to X_u \ell \bar{\nu}$: 3-body final state, appreciable rate in region 3), where hadronic final state not jet-like

  E.g., $m_X^2 < m_D^2$ does not imply $p_X^+ \ll p_X^-$

- Existing approaches based on theory in one region, extrapolated / modeled to rest

Past theoretical approaches

- BLNP [Bosch *et al.*] — based on SCET region
  - factorization & resummation in shape function region treated correctly
  - crossing into local OPE region not model independent
  - tied to "shape function" scheme
- DGE [Andersen & Gardi] — based on SCET region + perturbative model for the SF
  - SCET region treated correctly; motivated by renormalon resummation
- GGOU [Gambino *et al.*] — based on local OPE region + SF smearing
  - no resummation in SCET region
  - tied to "kinetic" scheme
- BLL [Bauer, ZL, Luke] — based on local OPE at large q^2 (but expansion scale is smaller)
  - combine q^2 and m_X cuts, such that SF effect is kept small
- Shape function independent relations [Leibovich, Low, Rothstein; Hoang, ZL, Luke; Lange, Neubert, Paz; Lange]
  - beautiful at leading order, less so when $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ included

Sources of theoretical uncertainties

1. Perturbative corrections: $\alpha_s(\sqrt{\Lambda_{\rm QCD} m_b}) \sim \alpha_s(m_c)$ for any kinematic cut that eliminates $B \to X_c \ell \bar\nu$ [both for $q^2 > (m_B - m_D)^2$ and for shape fn regions]
2. Weak annihilation: $\lesssim 3\%$ in rate, size of enhancement in shape fn region unclear How it populates triple differential rate
3. Other nonperturbative corrections: $\Lambda_{\rm QCD}/m_b$ in shape fn regions
4. Uncertainty related to fixed functional forms used to model shape function
5. No fully correct treatment in all regions of the phase space

In my opinion, the uncertainties related to 3–5 are currently underestimated

- Our goal is to address 2–5, and develop a fitter (SIMBA)

Even if measurement across much of phase space is possible [Belle, arXiv:0907.0379]
it will be beneficial to use multiple measurements to constrain all sources of errors

Our strategy towards precision $|V_{ub}|$

- To combine all phase space regions: do not expand Λ/p_X^+ nor p_X^+/p_X^- (nontrivial)

  [ZL, Stewart, Tackmann, to appear]

- Want to have: result accurate to NNLL and $\Lambda_{\rm QCD}/m_b$ in regions 1) – 2) and to order $\alpha_s^2\beta_0$ and $\Lambda_{\rm QCD}^2/m_b^2$ when phase space limits are in region 3)

- Incorporate all available constraints on shape function
  - Perturbative constraints (perturbative tail and RGE)
  - Moment constraints (m_b, λ_1, etc., from $B \to X_c \ell\bar\nu$)
  - Shape information ($B \to X_s\gamma$ and $B \to X_u \ell\bar\nu$ spectra)

- Repeat strategy successful for inclusive $|V_{cb}|$
  - Global fit to all available data
  - Simultaneously determine $|V_{ub}|$ & hadronic param's (m_b, shape fn's, WA, etc.)

The shape function (b quark PDF in B)

- The shape function $S(\omega, \mu)$ contains nonperturbative physics and obeys a RGE

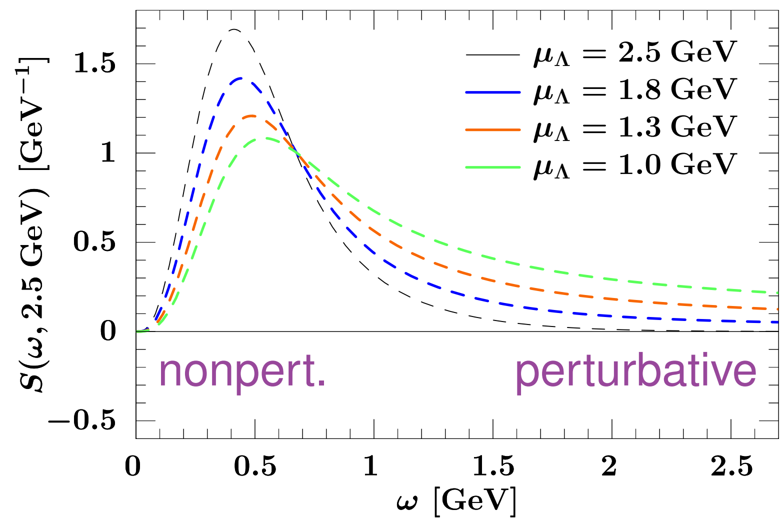
If $S(\omega, \mu_\Lambda)$ has exponentially small tail, but RGE running gives a long tail and divergent moments

[Balzereit, Mannel, Kilian]

$$S(\omega, \mu_i) = \int \mathrm{d}\omega' \, U_S(\omega - \omega', \mu_i, \mu_\Lambda) \, S(\omega', \mu_\Lambda)$$

Constraint: moments (OPE) + $B \to X_s \gamma$ shape

How to combine these?



Model $\left\{ S \right.$ (dash) run to $2.5\mathrm{GeV}$

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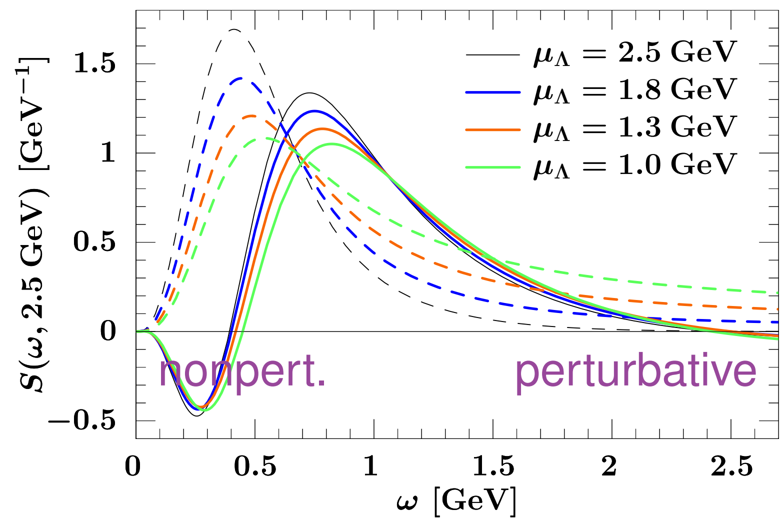
How to combine these?

Derive: [ZL, Stewart, Tackmann, 0807.1926]

$$S(\omega, \mu_\Lambda) = \int \mathrm{d}k\, C_0(\omega - k, \mu_\Lambda) F(k)$$



Model $\begin{cases} S & \text{(dash)} \\ F & \text{(solid)} \end{cases}$ run to 2.5GeV

– Consistent setup at any order, in any scheme

– Stable results for varying μ_Λ
(SF modeling scale, must be part of uncert.)

– Similar to how all matrix elements are defined
e.g., $B_K(\mu) = \widehat{B}_K\,[\alpha_s(\mu)]^{2/9}(1 + \ldots)$

- Consistent to impose moment constraints on $F(k)$, but not on $S(\omega, \mu_\Lambda)$ w/o cutoff

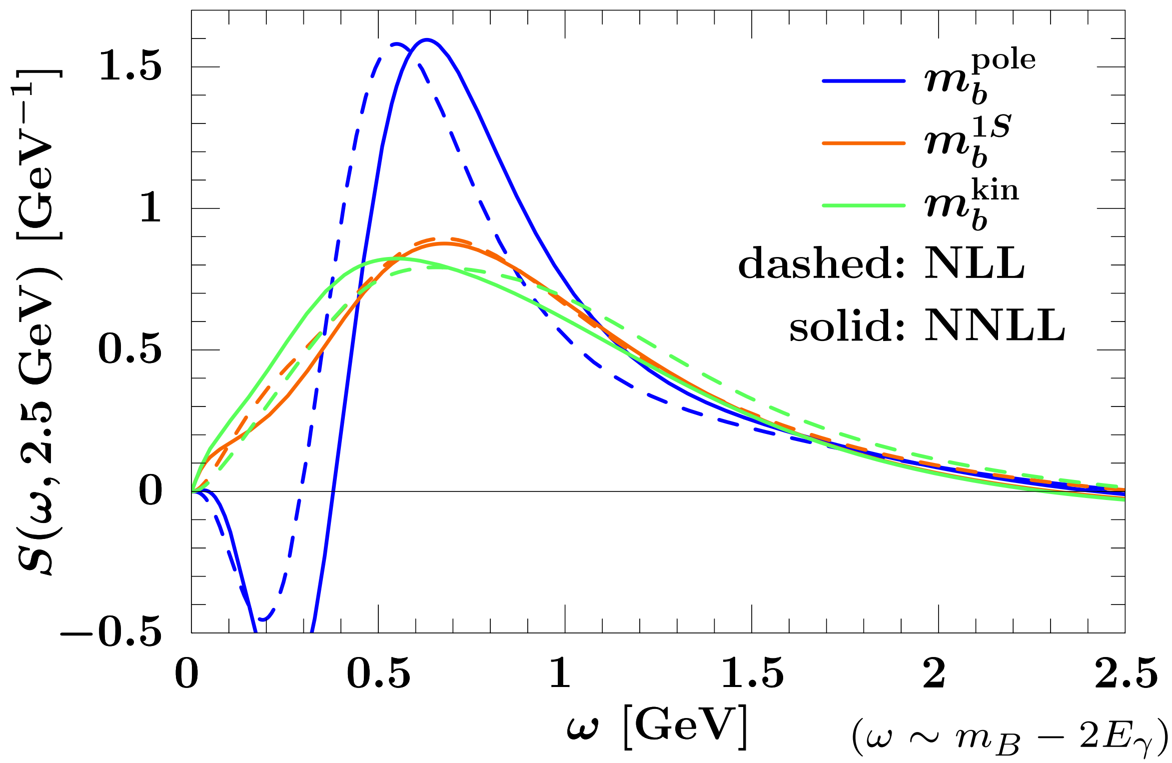
Changing schemes: m_b

- Converting results to a short distance mass scheme removes dip at small ω:
- Want to define short distance (hatted) quantities such that:

$$S(\omega) = \int \mathrm{d}k\, C_0(\omega - k)\, F(k)$$
$$= \int \mathrm{d}k\, \widehat{C}_0(\omega - k)\, \widehat{F}(k)$$

Switch from pole to short distance scheme:

$$m_b = \widehat{m}_b + \delta m_b$$
$$\lambda_1 = \widehat{\lambda}_1 + \delta\lambda_1$$



$(\omega \sim m_B - 2E_\gamma)$

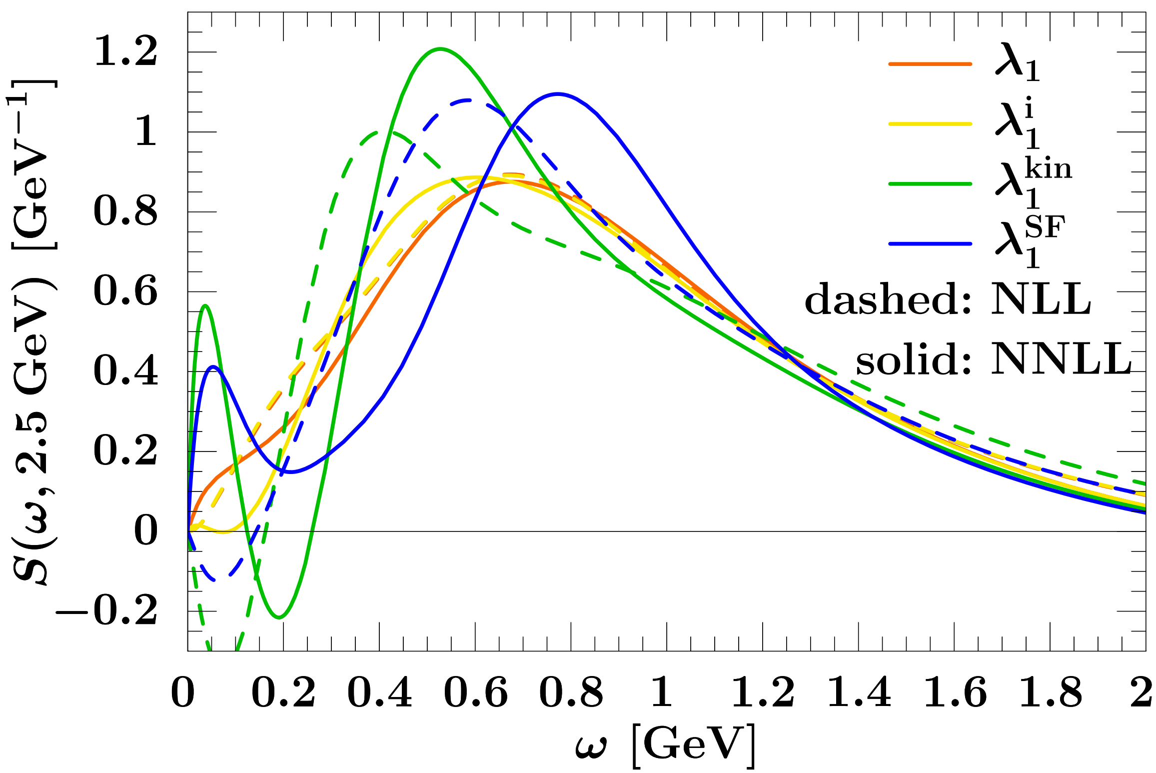
$$\widehat{C}_0(\omega) = C_0(\omega + \delta m_b) - \frac{\delta\lambda_1}{6}\frac{\mathrm{d}^2}{\mathrm{d}\omega^2}C_0(\omega) = \left[1 + \delta m_b \frac{\mathrm{d}}{\mathrm{d}\omega} + \left(\frac{(\delta m_b)^2}{2} - \frac{\delta\lambda_1}{6}\right)\frac{\mathrm{d}^2}{\mathrm{d}\omega^2}\right]C_0(\omega)$$

- Can use any short distance mass scheme ($1S$, kinetic, PS, shape function, ...)

Changing schemes: λ_1

- "Invisible" renormalon in λ_1 at $\mathcal{O}(\alpha_s^2)$? $\Rightarrow$ kinetic and SF schemes for λ_1
- Introduce invisible scheme: $\lambda_1^{\rm i} = \lambda_1 - 0\alpha_s - R^2 \dfrac{\alpha_s^2(\mu)}{\pi^2} \dfrac{C_F C_A}{4} \left(\dfrac{\pi^2}{3} - 1\right) \quad (R = 1\,\text{GeV})$
- Both kinetic and shape function scheme definitions ($\mu_\pi^2 \equiv -\lambda_1^{\rm kin}$, $\lambda_1^{\rm SF}$) over-subtract;

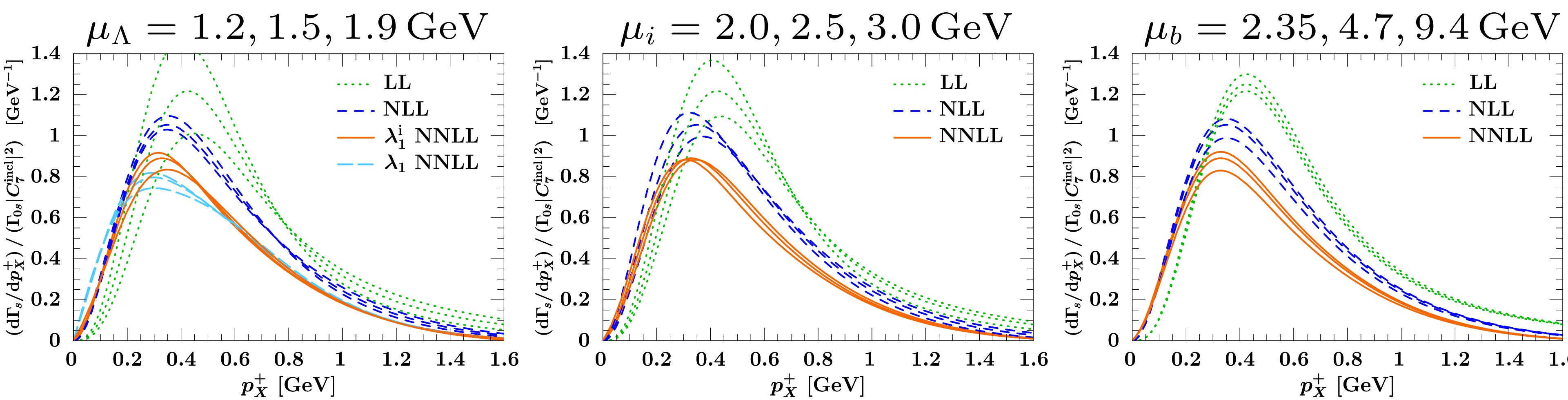
  ... similar to $\overline{m}_b(\overline{m}_b)$ issues



(Curves use $m_b^{1S,\,\rm kin,\,SF}$, respectively)

Scale (in)dependence of $B \to X_s\gamma$ spectrum

- Dependence on 3 scales in the problem can be handled appropriately:



$$\frac{\mathrm{d}\Gamma_s}{\mathrm{d}p_X^+} = \Gamma_{0s}\, H_s(p_X^+, \mu_b)\, U_H(m_b, \mu_b, \mu_i) \int \mathrm{d}k\, \widehat{P}(m_b, k, \mu_i)\, \widehat{F}(p_X^+ - k) \qquad \left(p_X^+ = m_B - 2E_\gamma\right)$$

$\widehat{P}, \widehat{F}$ indicate use of short distance schemes: m_b^{1S} and λ_1^{i}

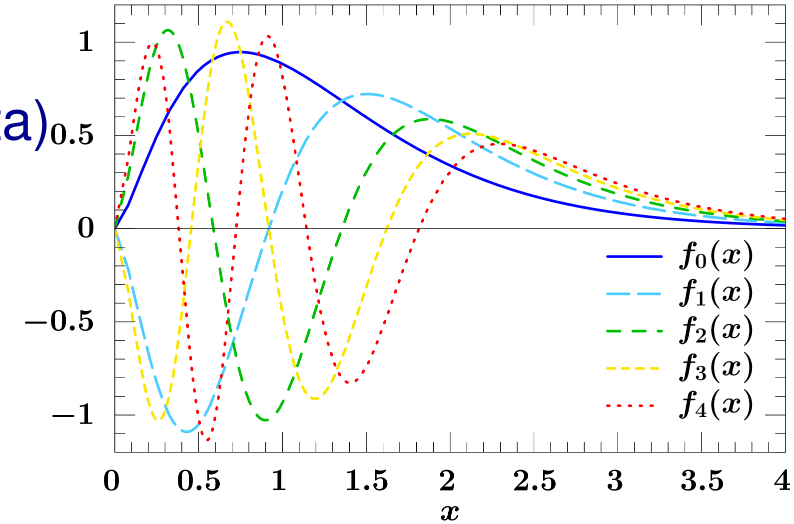
- In other approaches, using models for $S(\omega, \mu_\Lambda)$ run up to μ_i, dependence on μ_Λ ignored so far, but it must be considered an uncertainty $\Rightarrow$ This is how to solve it

Designer orthonormal functions

- Devise suitable orthonormal basis functions
  (earlier: fit parameters of model functions to data)

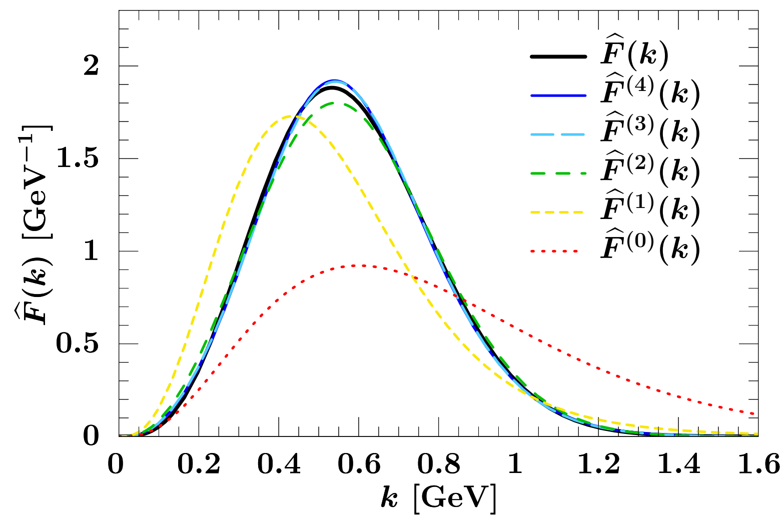
  $\widehat{F}(\lambda x) = \frac{1}{\lambda}\left[\sum c_n f_n(x)\right]^2$, n th moment $\sim \Lambda_{\rm QCD}^n$

  $f_n(x) \sim P_n[y(x)] \leftarrow$ Legendre polynomials



- Approximating a model shape function

  Better to add a new term in an orthonormal basis than a new parameter to a model:
  – less parameter correlations
  – errors easier to quantify



"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk." (John von Neumann)

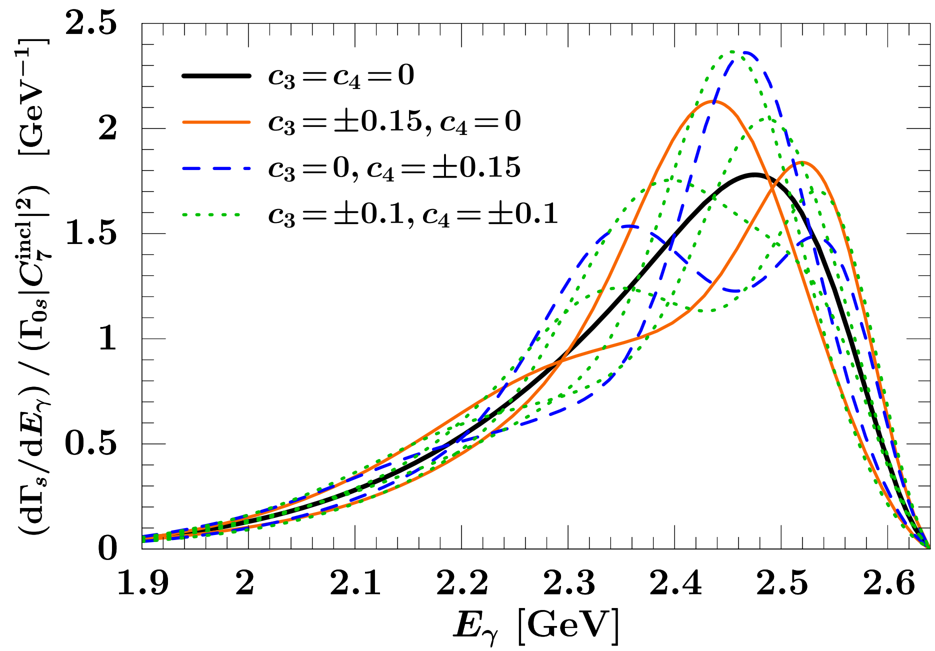
The $B \to X_s\gamma$ spectrum again

- 9 models: same 0th, 1st, 2nd moments

  Including all NNLL contributions, find:

  – Shape in peak region not determined at all by first few moments

  – Smaller shape function uncertainty for $E_\gamma \lesssim 2.1\,\mathrm{GeV}$ than earlier studies



- Not shown in this plot: subleading shape functions

  subleading corrections not in $C_7^{\rm incl}(0)$

  kinematic powercorrections

  boost to $\Upsilon(4S)$ frame

- Same analysis can also be used for: $B \to X_s \ell^+\ell^-$, $|V_{ub}|$

Global fit to charmless inclusive B decays

- Strategy for global fit: $\widehat{F}(k)$ enters the spectra linearly $\Rightarrow$ can calculate independently the contribution of $f_m\, f_n$ in the expansion of $\widehat{F}(k)$:

$$\mathrm{d}\Gamma = \sum \underbrace{c_m\, c_n}_{\text{fit}}\ \underbrace{\mathrm{d}\Gamma_{mn}}_{\text{compute}}$$

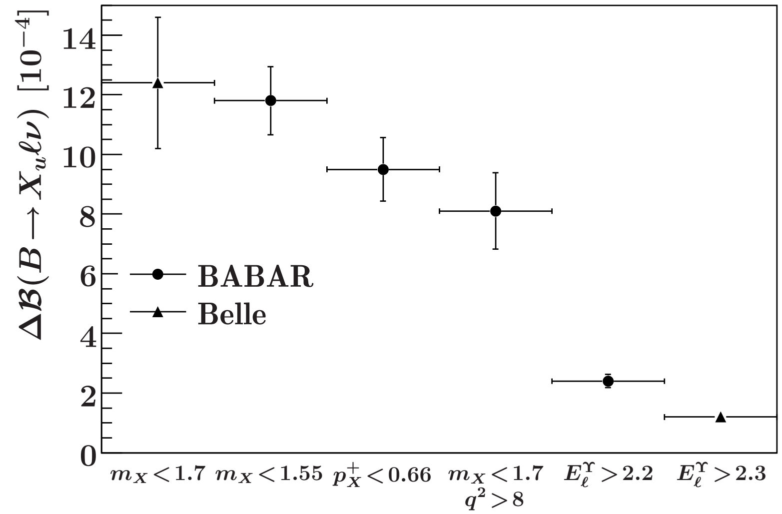
$$\mathrm{d}\Gamma_{mn} = \Gamma_0\, H(p_X^{\pm}) \int_0^{p_X^+} \mathrm{d}k\, \frac{\widehat{P}(p^-, k)}{\lambda}\ \underbrace{f_m\!\left(\frac{p_X^+ - k}{\lambda}\right) f_n\!\left(\frac{p_X^+ - k}{\lambda}\right)}_{\text{basis functions}}$$

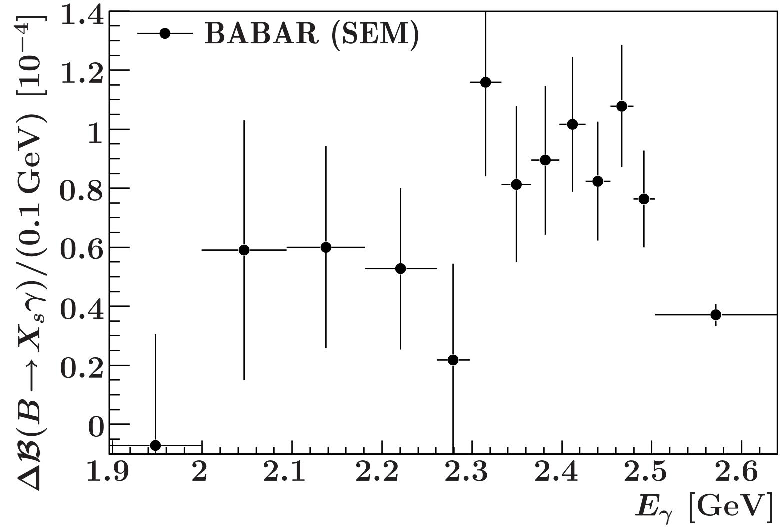
Fit the c_i coefficients from all measured (binned) spectra (similar to $|V_{cb}|$ fit)

- What SIMBA hopes to achieve:
  - Correlation and error propagation of SF uncertainties
  - Simultaneous fit using all available information
  - Realistic estimate of model uncertainties (fit parameters $c_{0,\ldots,N}$ constrained by data; vary N, the number of orthonormal basis functions in fit)

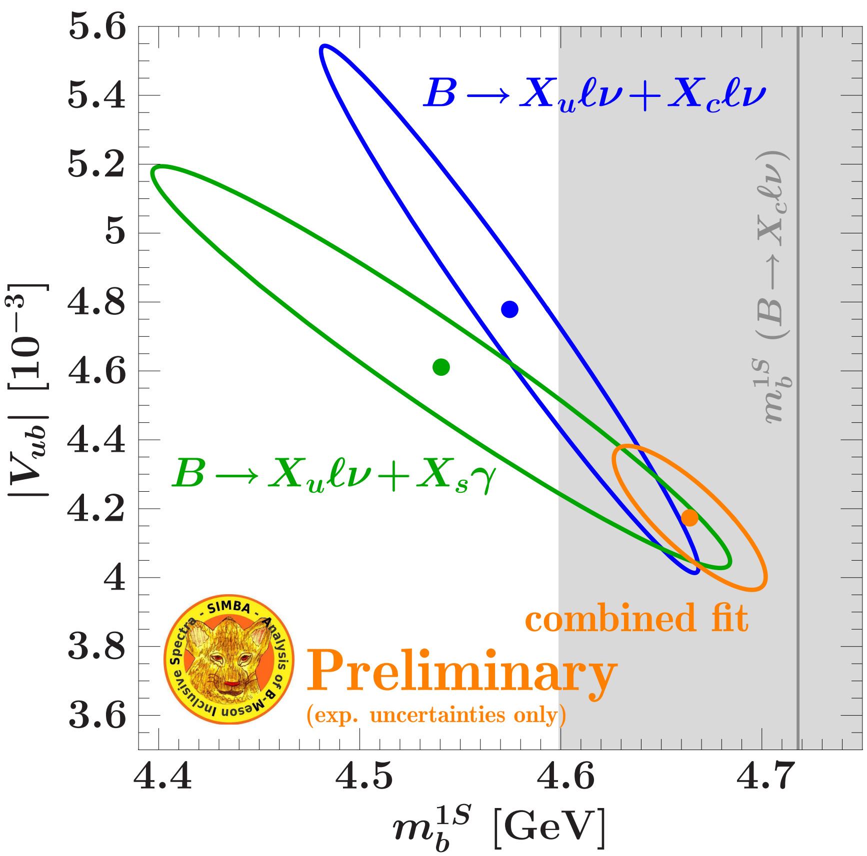
Proof of concept — a preliminary SIMBA fit

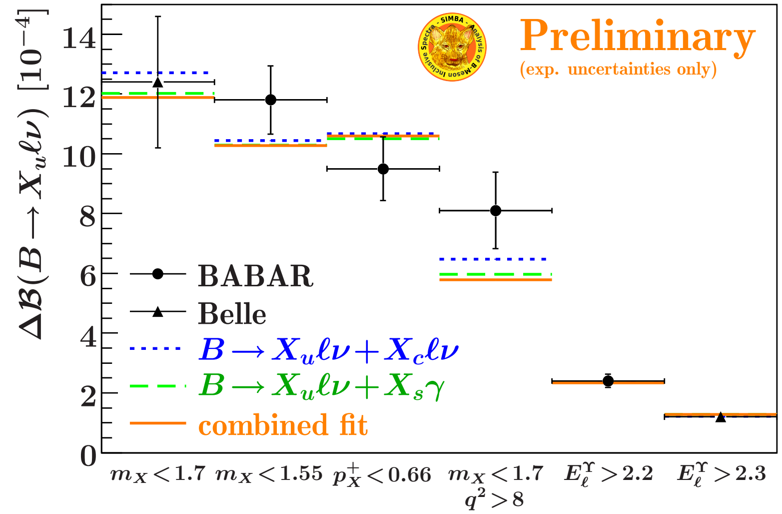
- SIMBA [Bernlochner, Lacker, ZL, Stewart, Tackmann, Tackmann, in progress]
- $B \to X_u \ell \bar{\nu}$ hadronic tag
  - BaBar $m_X,\ m_X - q^2,\ p_X^+$
  - Belle m_X
- $B \to X_u \ell \bar{\nu}$ lepton endpoint
  - BaBar $E_\ell^\Upsilon > 2.2\,\mathrm{GeV}$
  - Belle $E_\ell^\Upsilon > 2.3\,\mathrm{GeV}$
- $B \to X_s \gamma$ spectra
  - BaBar sum over exclusive
  - BaBar hadronic tag (not shown)
- m_b^{1S}, λ_1 from $B \to X_c \ell \bar{\nu}$
  - Belle $1S$ scheme fit result

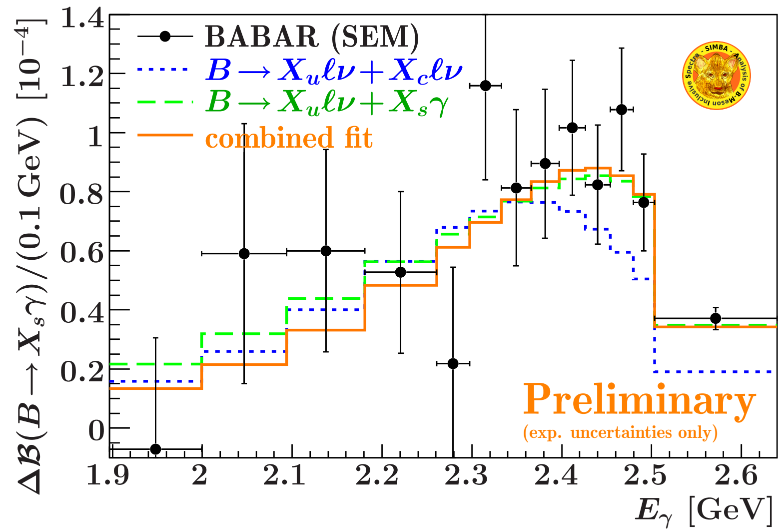




Proof of concept — a preliminary SIMBA fit







- E_γ spectrum is off without $B \to X_s\gamma$ in the fit
- Significant improvement from combining the $B \to X_s\gamma$ data with $B \to X_c \ell\bar{\nu}$ moments

Conclusions

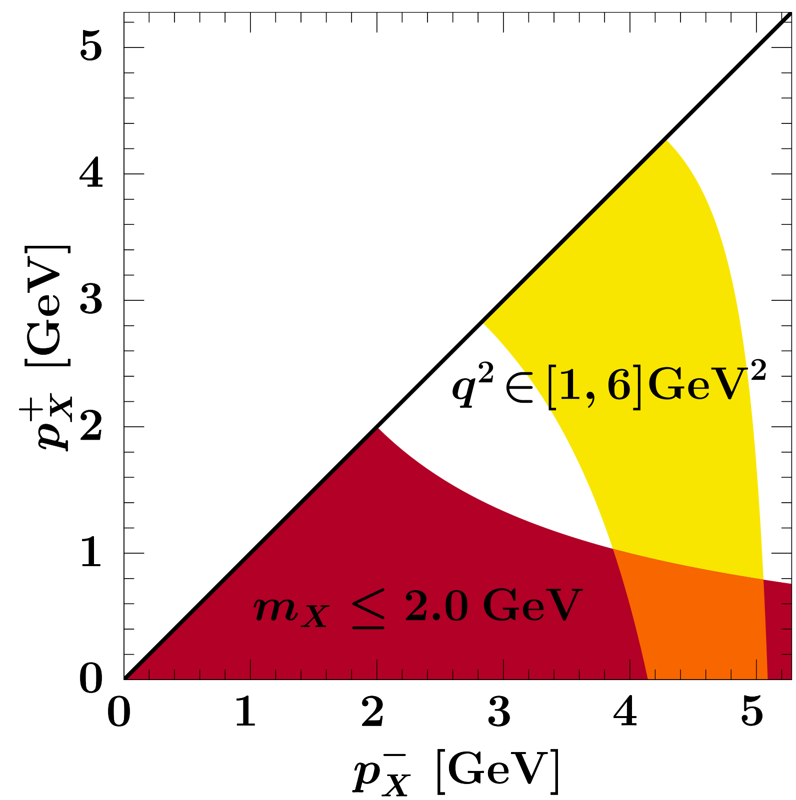
- Improving accuracy of $|V_{xb}|$ will remain important to constrain new physics (Current situation unsettled, PDG in 2008 inflated errors for the first time)
- Qualitatively better inclusive $|V_{ub}|$ analysis possible than those implemented so far
  - Modeling $F(k)$ instead of $S(\omega, \mu)$
  - Designer orthonormal functions — reduce role of shape function modeling
  - Fully consistent combination of all phase space regions
  - Decouple SF shape variation from m_b variation
- Developments will allow combining all pieces of data with tractable uncertainties
  - Consistently combine $B \to X_u \ell \bar{\nu}$, $B \to X_s \gamma$, $B \to X_c \ell \bar{\nu}$ data to constrain SFs
  - Inclusive $|V_{cb}|$ uses a combined fit; clearly the right method for $|V_{ub}|$ as well
- $|V_{ub}|$ is tricky: to draw conclusions about new physics, we'll want ≥ 2 extractions with different uncertainties to agree well (inclusive, exclusive, leptonic)

Backup slides

Regions of $B \to X_s \ell^+ \ell^-$ phase space

- Main difference in $B \to X_s \ell^+ \ell^-$: only regions 1) – 2) are relevant for $q^2 < m_\psi^2$

  The $q^2 > m_{\psi'}^2$ region is like large-q^2 in $B \to X_u \ell \bar{\nu}$



The BLNP approach

- Treated factorization & resummation in shape function region correctly
- Use fixed functional forms to model shape function (similar to PDF's)
- Analysis tied to shape function scheme for m_b, λ_1 (One scheme for each approach)
- Need for "tail gluing" to get correct perturbative tail (other approaches don't even do this [DGE, GGOU, ADFR])

$\Rightarrow$ Hard to assess uncertainties

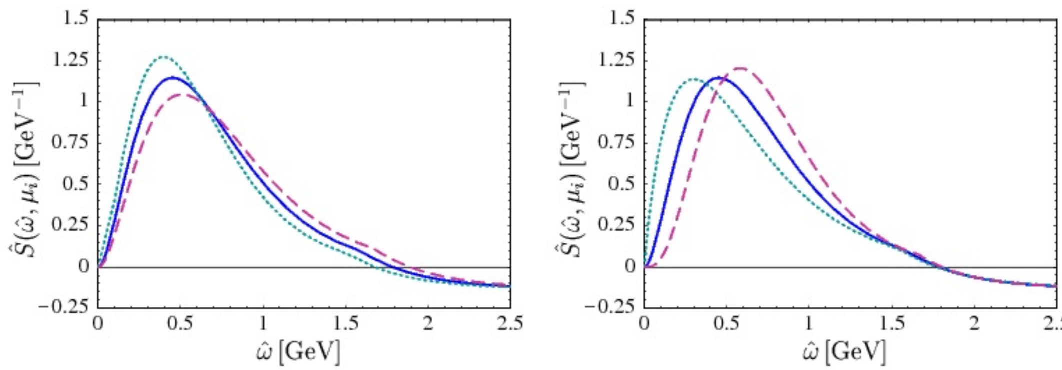


Figure 6: Various models for the shape function at the intermediate scale $\mu_i = 1.5$ GeV, corresponding to different parameter settings in Table 1. Left: Functions S1, S5, S9 with "correlated" parameter variations. Right: Functions S3, S5, S7 with "anti-correlated" parameter variations.

[Bosch, Lange, Neubert, Paz]

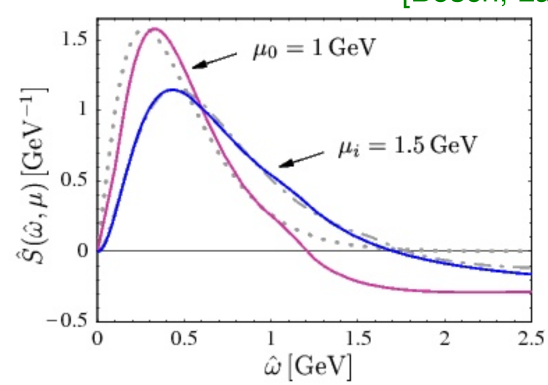


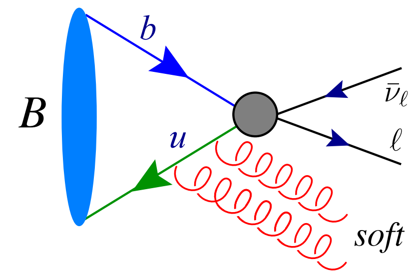
Figure 7: Renormalization-group evolution of a model shape function from a low scale μ_0 (sharply peaked solid curve) to the intermediate scale μ_i (broad solid curve). See the text for an explanation of the other curves.

Weak annihilation

- Old story: $16\pi^2 \frac{\Lambda_{\rm QCD}^3}{m_b^3}\, \varepsilon$ in total rate, smeared around $\delta(q^2 - m_B^2)\, \delta(E_\ell - \frac{m_B}{2})$

  Guesstimate: $\lesssim 3\%$ of $B \to X_u \ell \bar\nu$ rate

  At large E_ℓ: enhanced by $\frac{m_b}{\Lambda_{\rm QCD}} \Rightarrow$ sizable uncertainty

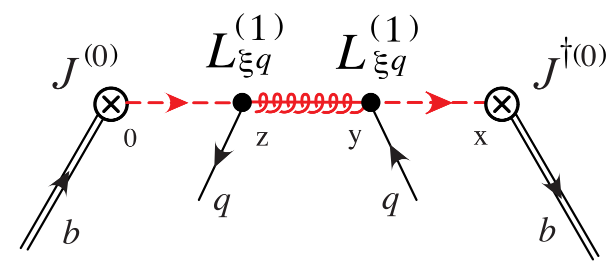


- More recent: $4\pi\alpha_s \frac{\Lambda_{\rm QCD}^3}{m_b^3}\, \varepsilon$ in total rate, smeared in usual shape fn. region

  Role / presence of $4\pi\alpha_s$ and ε factors argued

  In shape fn. regions: enhanced by $\frac{m_b^2}{\Lambda_{\rm QCD}^2}$

  Can be absorbed into other subleading shape fn.'s



[Lee & Stewart; Bosch, Neubert, Paz]

- Need to constrain it directly from data by comparing D^0 vs. D_s semileptonic widths, or $|V_{ub}|$ from $B^\pm$ vs. B^0 decay

Derivation of the magic formula (1)

- The shape function is the matrix element of a nonlocal operator:

$$S(\omega,\mu) = \langle B|\, \underbrace{\bar b_v\, \delta(iD_+ - \delta + \omega)\, b_v}_{O_0(\omega,\mu)} \,|B\rangle, \qquad \delta = m_B - m_b$$

Integrated over a large enough region, $0 \le \omega \le \Lambda$, one can expand O_0 as

$$O_0(\omega,\mu) = \sum C_n(\omega,\mu)\, \underbrace{\bar b_v\, (iD_+ - \delta)^n\, b_v}_{Q_n} + \ldots = \sum C_n(\omega-\delta,\mu)\, \underbrace{\bar b_v\, (iD_+)^n\, b_v}_{\widetilde Q_n} + \ldots$$

The C_n are the same for Q_n and $\widetilde Q_n$ (since O_0 only depends on $\omega - \delta$)

- Matching: $\langle b_v|O_0(\omega{+}\delta,\mu)|b_v\rangle = \sum C_n(\omega,\mu)\, \langle b_v|\widetilde Q_n|b_v\rangle = C_0(\omega,\mu), \quad \langle b_v|\widetilde Q_n|b_v\rangle = \delta_{0n}$

$$\langle b_v(k_+)|O_0(\omega+\delta,\mu)|b_v(k_+)\rangle = C_0(\omega+k_+,\mu) = \sum \frac{k_+^n}{n!}\, \frac{\mathrm{d}^n C_0(\omega,\mu)}{\mathrm{d}\omega^n}$$

$$\langle b_v(k_+)|O_0(\omega+\delta,\mu)|b_v(k_+)\rangle = \sum C_n(\omega,\mu) \langle b_v|\widetilde Q_n|b_v\rangle = \sum C_n(\omega,\mu)\, k_+^n$$

- Comparing last two lines: $C_n(\omega,\mu) = \dfrac{1}{n!}\, \dfrac{\mathrm{d}^n C_0(\omega,\mu)}{\mathrm{d}\omega^n}$ [Bauer & Manohar]

Derivation of the magic formula (2)

- Define the nonperturbative function $F(k)$ by: [ZL, Stewart, Tackmann; Lee, ZL, Stewart, Tackmann]

$$S(\omega, \mu_\Lambda) = \int \mathrm{d}k\, C_0(\omega - k, \mu_\Lambda)\, F(k), \qquad C_0(\omega, \mu) = \langle b_v | O_0(\omega + \delta, \mu) | b_v \rangle$$

uniquely defines $F(k)$: $\widetilde{F}(y) = \widetilde{S}(y, \mu) / \widetilde{C}_0(y, \mu)$

- Expand in k: $S(\omega, \mu) = \sum_n \frac{1}{n!} \frac{\mathrm{d}^n C_0(\omega, \mu)}{\mathrm{d}\omega^n} \int \mathrm{d}k\, (-k)^n F(k)$

Compare with previous page $\Rightarrow \int \mathrm{d}k\, k^n F(k) = (-1)^n \langle B | Q_n | B \rangle$

$$\langle B | Q_0 | B \rangle = 1\,, \quad \langle B | Q_1 | B \rangle = -\delta\,, \quad \langle B | Q_2 | B \rangle = -\frac{\lambda_1}{3} + \delta^2$$

More complicated situation for higher moments, so stop here

- This treatment is fully consistent with the OPE

One-page derivation of the magic formula

- The shape function is the B meson matrix element of a nonlocal operator:

$$S(\omega,\mu) = \langle B|\underbrace{\bar b_v\,\delta(iD_+ - \delta + \omega)\,b_v}_{O_0(\omega,\mu)}|B\rangle, \qquad \delta = m_B - m_b$$

Integrated over a large enough region, $0 \le \omega \le \Lambda$, one can expand O_0 as

$$O_0(\omega,\mu) = \sum C_n(\omega,\mu)\,\underbrace{\bar b_v\,(iD_+ - \delta)^n\,b_v}_{Q_n} + \ldots = \sum C_n(\omega-\delta,\mu)\,\underbrace{\bar b_v\,(iD_+)^n\,b_v}_{\widetilde Q_n} + \ldots$$

C_n same for Q_n and $\widetilde Q_n$ (since O_0 only depends on $\omega-\delta$), determined by matching

Evaluating $\langle O_0\rangle_{b_v(k)}$, can show using RPI: $C_n(\omega,\mu) = \dfrac{1}{n!}\,\dfrac{\mathrm{d}^n C_0(\omega,\mu)}{\mathrm{d}\omega^n}$ [Bauer & Manohar]

- Define the nonperturbative function $F(k)$ by: [ZL, Stewart, Tackmann; Lee, ZL, Stewart, Tackmann]

$$S(\omega,\mu_\Lambda) = \int \mathrm{d}k\, C_0(\omega-k,\mu_\Lambda)\,F(k), \qquad C_0(\omega,\mu) = \langle b_v|\,O_0(\omega+\delta,\mu)\,|b_v\rangle$$

Expand in k, compare: $\int \mathrm{d}k\,k^n F(k) = (-1)^n\,\langle Q_n\rangle_B$ $\Rightarrow$ fully consistent with OPE